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معهد البحوث العلمية
مركز بحوث العلوم التطبيقية

سلسلة بحوث العلوم التطبيقية

الحمل الحراري في ثلاث طبقات أفقية لمقارنة نتائج الحلول بواسطة الرتبة الأولى والرتبة الثانية من طريقة تشيبشيف CHEBYSHEV الطيفية

د. عبدالفتاح بن قاري بخاري

الطالبة : هنادي محمد بنجر

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طريقة بواسطة الحاسب الآلي لتخطيط شبكة صرف لمياه الأمطار باستخدام منظومة جينية

الدكتور/ سعود بن محمد عبد الله مغربي

قسم العلوم الرياضية / كلية العلوم التطبيقية

ص. ب ٦٦٤٨

مكة المكرمة – المملكة العربية السعودية

الملخص

يتم تجميع مياه الأمطار المتدفقة في بالوعات ، ويلزم نقلها خلال شبكة صرف إلى نقطة التخلص منها. وفي نظام الصرف المتدفق الذي يخدم منطقة سكنية تدخل مياه الأمطار من خلال فتحات شبكية موضوعة على طول الطرق، ويتم نقلها في اتجاه بالوعات توضع عادة في النهايات. وهكذا، تقع البالوعات عند وصلات المواسير ، وتمثل نقط الدخول لهذا الجزء من نظام صرف مياه الأمطار، النظام الفرعي، الذي يحمل الصرف إلى نقطة التخلص منه. وفي التطبيق العادي تكون هذه الشبكة في شكل شجرة ، مع تدفق المياه عند عقد التقاء الشجرة. والمشكلة تتمثل في القرار الخاص بتوصيل أي البالوعات بمواسير وقطر الماسورة المستخدم ، بهدف تقليل التكلفة الكلية. ويشار إلى هذا بـ " مشكلة تخطيط الشبكة "، وهذا هو ما تهتم به مشكلة هذا البحث.

فالهدف من هذا البحث هو إيجاد الحل الأمثل لمشكلة تخطيط شبكة صرف مياه الأمطار باستخدام منظومة جينية (وراثية). والحل الأمثل هو الأقل تكلفة. وتهدف المنظومات الجينية إلى حل مشاكل الأفضلية باستخدام بحث توالدي سكاني مبني على ميكانيكية الاختيار الطبيعي.

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equations. R & M 1799, Aeronautical Research Council, London.

Gardner, D. R., Trogdon, S. A. & Douglass, R. W. 1989. A modified tau spectral method that eliminates spurious eigenvalues. Journal of Computational Physics. 80: 137-167.

Kantorovic, L. V. 1934. On a new method of approximate solution of partial differential equations. Doklady Akademii Nauk SSSR 4: 532-536 (in Russian).

Lanczos, C. 1938. Trigonometric interpolation of empirical and analytical functions. Journal of Mathematical Physics. 17: 123-199.

Orszag, S.A. 1969. Numerical methods for the simulation of turbulence. Physics of Fluids, Suppl. II, 12: 250-257.

Orszag, S.A.1970. Transform method for calculation of vector coupled sums: Application to the spectral form of the vorticity equation .Journal of the Atmospheric Sciences. 27: 890-895.

Orszag, S. A. 1971a. Accurate solution of the Orr-Sommer field stability equation. Journal of Fluid Mechanics.50: 689-703.

Orszag, S. A. 1971b. Galerkin approximations to flows with slabs, spheres and cylinders. Physical Review Letters. 26: 1100-1103.

Orszag, S. A. & Kells, L. C. 1980. Transition to turbulence in plane Poiseuille flow and plane Couette flow. Journal of Fluid Mechanics. 96: 159-205.

Silberman, I. 1954. Planetary waves in the atmosphere. Journal of the Meteorology. 11: 27-34.

Slater, J. C. 1934. Electronic energy bands in metal. Physical Review. 45: 794-801.

Villadsen, J. V. & Stewart, W. E. 1967. Solution of boundary value problems by orthogonal collocation. Chemical Engineering Science. 22: 1483-1501.

Wright, K. 1964. Chebyshev collocation methods for ordinary differential equations. Computer Journal. 6: 358-365.

References

Beavers, G. S. & Joseph, D. D. 1967. Boundary conditions at a naturally permeable wall. Journal of Fluid Mechanics. 30: 197-207.

Bridges, T. J. & Morris, P. J. 1984a. Differential eigenvalue problems in which the parameter appears nonlinearly. Journal of Computational Physics. 55: 437-460.

Bridges, T. J. & Morris, P. J. 1984b. Spectral calculations of the spatial stability of non-parallel boundary layers. AIAA Pap. No. 84-0437.

Bukhari, A. F. 1997. A spectral eigenvalue method for multi-layered continuua. Ph. D. thesis, University of Glasgow.

Chaves, T. & Ortiz, E. L. 1968. On the numerical solution of two-point boundary value problems for linear differential equations. Zeitschrift fur Angewandte Mathematik und Mechanik. 48: 415-418.

Chen, F. & Chen, C. F. 1988. Onset of finger convection in a horizontal porous layer underlying a fluid layer. Journal of Heat Transfer. 110: 403-409.

Clenshaw, C. W. 1957. The numerical solution of linear differential equations in Chebyshev series. Proceedings of the Cambridge Philosophical Society. 53: 134-149.

Clenshaw, C. W. & Norton, H. J. 1963. The solution of nonlinear ordinary differential equations in Chebyshev series. Computer Journal. 6: 88-92.

Eliasen, E., Machenhauer, B. & Rasmussen, E. 1970. On a numerical method for integration of the hydrodynamical equations with a spectral representation of the horizontal fields Rep. No. 2 Institut for Teoretisk Meteorologi, University of Copenhagen.

Fox, L. 1962. Chebyshev methods for ordinary differential equations. Computer Journal. 4: 318-331.

Fox, L. & Parker, I. B. 1968. Chebyshev polynomials in numerical analyses. Oxford University Press, London.

Frazer, R.A., Jones, W. P. & Skan, S. W.. 1937. Approximation to functions and to the solution of differential

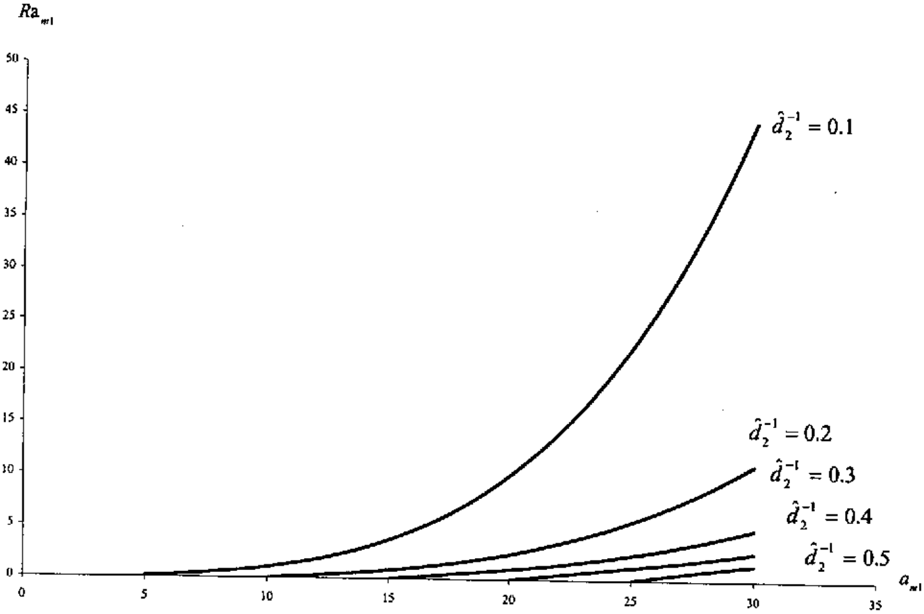

Fig. 4. The relation between a_{m1} and Ra_{m1} by using the first order of Chebyshev spectral method when $\hat{d}_1^{-1} = 0.5$, $M = 20$ and $M = 40$.

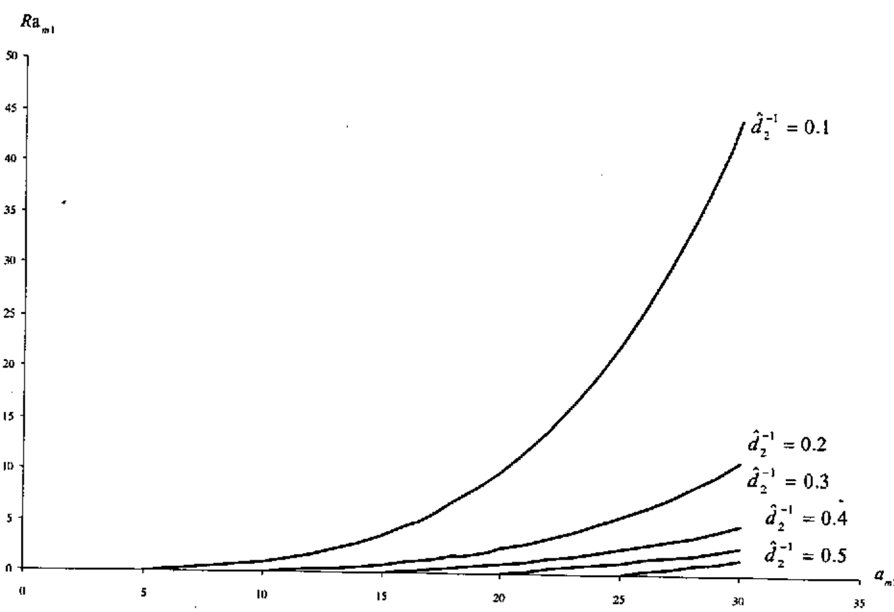

Fig. 5. The relation between a_{m1} and Ra_{m1} by using the second order of Chebyshev spectral method when $\hat{d}_1^{-1} = 0.5$, $M = 20$ and $M = 40$.

Table 9. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $d_1^{''} = 0.5$ and $d_2^{''} = 0.4$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
20	0.000002209008	0.000002209010	2.00E-12	0.000002208955	0.000002208847	1.08E-10	5.30E-11	1.63E-10
21	0.264866244021	0.264866243982	3.90E-11	0.264866244217	0.264866243947	2.70E-10	1.96E-10	3.50E-11
22	0.515250905135	0.515250905110	2.50E-11	0.515250905639	0.515250904656	9.83E-10	5.04E-10	4.54E-10
23	0.756211095951	0.756211096001	5.00E-11	0.756211097003	0.756211095996	1.01E-09	1.05E-09	5.00E-12
24	0.994286169394	0.994286169402	8.00E-12	0.994286170914	0.994286169619	1.29E-09	1.52E-09	2.17E-10
25	1.236764817626	1.236764817563	6.30E-11	1.236764819841	1.236764817515	2.33E-09	2.21E-09	4.80E-11
26	1.490477997155	1.490477997074	8.10E-11	1.490478000501	1.490477997009	3.49E-09	3.35E-09	6.50E-11
27	1.761151345647	1.761151345530	1.17E-10	1.761151350728	1.761151345551	5.18E-09	5.08E-09	2.10E-11
28	2.053415684654	2.053415684440	2.14E-10	2.053415692223	2.053415684327	7.90E-09	7.57E-09	1.13E-10
29	2.371087665552	2.371087665245	3.07E-10	2.371087676741	2.371087665354	1.14E-08	1.12E-08	1.09E-10
30	2.717454668656	2.717454668257	3.99E-10	2.717454684942	2.717454668115	1.68E-08	1.63E-08	1.42E-10

Table 10. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $d_1^{''} = 0.5$ and $d_2^{''} = 0.5$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
25	0.000000036339	0.000000036338	1.00E-12	0.000000033492	0.000000036225	2.73E-09	2.85E-09	1.13E-10
26	0.332963643772	0.332963643772	0.00E+00	0.332963647535	0.332963643818	3.72E-09	3.76E-09	4.60E-11
27	0.649905230758	0.649905230736	2.20E-11	0.649905241317	0.649905230594	1.07E-08	1.06E-08	1.42E-10
28	0.953467909476	0.953467909420	5.60E-11	0.953467926210	0.953467909299	1.69E-08	1.67E-08	1.21E-10
29	1.247065657683	1.247065657543	1.40E-10	1.247065679194	1.247065657901	2.13E-08	2.15E-08	3.58E-10
30	1.535640952534	1.535640952163	3.71E-10	1.535640977137	1.535640952306	2.48E-08	2.46E-08	1.43E-10

Table 8. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 0.5$ and $\hat{d}_2^{-1} = 0.3$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
15	0.000104042989	0.000104042989	0.00E+00	0.000104042984	0.000104042644	3.40E-10	5.00E-12	3.45E-10
16	0.198709513786	0.198709513786	0.00E+00	0.198709513814	0.198709513705	1.09E-10	2.80E-11	8.10E-11
17	0.388769758382	0.388769758383	1.00E-12	0.388769758560	0.388769758370	1.90E-10	1.78E-10	1.30E-11
18	0.578885918263	0.578885918260	3.00E-12	0.578885918845	0.578885918180	6.65E-10	5.82E-10	8.00E-11
19	0.777102449282	0.777102449265	1.70E-11	0.777102450865	0.777102449271	1.59E-09	1.58E-09	6.00E-12
20	0.990259262975	0.990259262982	7.00E-12	0.990259266544	0.990259262982	3.56E-09	3.57E-09	0.00E+00
21	1.223999282636	1.223999282618	1.80E-11	1.223999289826	1.223999282611	7.21E-09	7.19E-09	7.00E-12
22	1.483077562223	1.483077562217	6.00E-12	1.483077575521	1.483077562281	1.32E-08	1.33E-08	6.40E-11
23	1.771683561568	1.771683561539	2.90E-11	1.771683584592	1.771683561461	2.31E-08	2.30E-08	7.80E-11
24	2.093682518017	2.093682518008	9.00E-12	2.093682556014	2.093682517832	3.82E-08	3.80E-08	1.76E-10
25	2.452776049831	2.452776049792	3.90E-11	2.452776110007	2.452776049798	6.02E-08	6.02E-08	6.00E-12
26	2.852604588384	2.852604588348	3.60E-11	2.852604680566	2.852604588204	9.24E-08	9.22E-08	1.44E-10
27	3.296812158995	3.296812158925	7.00E-11	3.296812296248	3.296812158909	1.37E-07	1.37E-07	1.60E-11
28	3.789087659206	3.789087658775	4.31E-10	3.789087858227	3.789087658549	2.00E-07	1.99E-07	2.26E-10
29	4.333191555370	4.333191555163	2.07E-10	4.333191838656	4.333191555186	2.83E-07	2.83E-07	2.30E-11
30	4.932973468646	4.932973468293	3.53E-10	4.932973863623	4.932973468421	3.95E-07	3.95E-07	1.28E-10

Table 7. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 0.5$ and $\hat{d}_2^{-1} = 0.2$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M=20$	Ra_{m1} when $M=40$	Accuracy	Ra_{m1} when $M=20$	Ra_{m1} when $M=40$	Accuracy	Accuracy for $M=20$	Accuracy for $M=40$
10	0.003295272825	0.003295272825	0.00E+00	0.003295272836	0.003295272879	4.30E-11	1.10E-11	5.40E-11
11	0.147429528913	0.147429528913	0.00E+00	0.147429529205	0.147429528937	2.68E-10	2.92E-10	2.40E-11
12	0.295929212486	0.295929212486	0.00E+00	0.295929213885	0.295929212529	1.36E-09	1.40E-09	4.30E-11
13	0.457359269910	0.457359269910	0.00E+00	0.457359274529	0.457359269884	4.64E-09	4.62E-09	2.60E-11
14	0.638831402683	0.638831402684	1.00E-12	0.638831415028	0.638831402675	1.24E-08	1.23E-08	9.00E-12
15	0.846590559331	0.846590559331	0.00E+00	0.846590587952	0.846590559355	2.86E-08	2.86E-08	2.40E-11
16	1.086419348409	1.086419348410	1.00E-12	1.086419408380	1.086419348457	5.99E-08	6.00E-08	4.70E-11
17	1.363903962868	1.363903962870	2.00E-12	1.363904079004	1.363903962840	1.16E-07	1.16E-07	3.00E-11
18	1.684604651226	1.684604651224	2.00E-12	1.684604862270	1.684604651101	2.11E-07	2.11E-07	1.23E-10
19	2.054164237915	2.054164237909	6.00E-12	2.054164601906	2.054164237860	3.64E-07	3.64E-07	4.90E-11
20	2.478377538969	2.478377538963	6.00E-12	2.478378139722	2.478377538981	6.01E-07	6.01E-07	1.80E-11
21	2.963236279537	2.963236279517	2.00E-11	2.963237234308	2.963236279633	9.55E-07	9.55E-07	1.16E-10
22	3.514958599594	3.514958599443	1.51E-10	3.514960067988	3.514958599519	1.47E-06	1.47E-06	7.60E-11
23	4.140008759310	4.140008759287	2.30E-11	4.140010953959	4.140008759078	2.19E-06	2.19E-06	2.09E-10
24	4.845110527052	4.845110527018	3.40E-11	4.845113724347	4.845110526928	3.20E-06	3.20E-06	9.00E-11
25	5.637256426083	5.637256425691	3.92E-10	5.637260978942	5.637256425700	4.55E-06	4.55E-06	9.00E-12
26	6.523714226877	6.523714226167	7.10E-10	6.523720578964	6.523714226375	6.35E-06	6.35E-06	2.08E-10
27	7.512031577500	7.512031576405	1.10E-09	7.512040277432	7.512031575969	8.70E-06	8.70E-06	4.36E-10
28	8.610039351922	8.610039350642	1.28E-09	8.610051069095	8.610039350656	1.17E-05	1.17E-05	1.40E-11
29	9.825854108589	9.825854105942	2.65E-09	9.825869647665	9.825854105805	1.55E-05	1.55E-05	1.37E-10
30	11.167879911110	11.167879907369	3.74E-09	11.167900231442	11.167879907181	2.03E-05	2.03E-05	1.88E-10

Table 6. Rayleigh number Ra_{m1} corresponding to wave number u_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 0.5$ and $\hat{d}_2^{-1} = 0.1$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M=20$	Ra_{m1} when $M=40$	Accuracy	Ra_{m1} when $M=20$	Ra_{m1} when $M=40$	Accuracy	Accuracy for $M=20$	Accuracy for $M=40$
5	0.084002581960	0.084002581960	0.00E+00	0.084002582623	0.084002581927	6.96E-10	6.63E-10	3.30E-11
6	0.217192071890	0.217192071890	0.00E+00	0.217192079922	0.217192071896	8.03E-09	8.03E-09	6.00E-12
7	0.371191245680	0.371191245680	0.00E+00	0.371191292306	0.371191245680	4.66E-08	4.66E-08	0.00E+00
8	0.557539456123	0.557539456118	5.00E-12	0.557539641609	0.557539456142	1.85E-07	1.85E-07	2.40E-11
9	0.787247585364	0.787247585362	2.00E-12	0.787248164491	0.787247585425	5.79E-07	5.79E-07	6.30E-11
10	1.071671634947	1.071671634934	1.30E-11	1.071673158124	1.071671634916	1.52E-06	1.52E-06	1.80E-11
11	1.422878513606	1.422878513537	6.90E-11	1.422882035097	1.422878513487	3.52E-06	3.52E-06	5.00E-11
12	1.853824222692	1.853824222421	2.71E-10	1.853831581275	1.853824222525	7.36E-06	7.36E-06	1.04E-10
13	2.378452110666	2.378452109739	9.27E-10	2.378466281060	2.378452109743	1.42E-05	1.42E-05	4.00E-12
14	3.011752330779	3.011752327945	2.83E-09	3.011777836982	3.011752327867	2.55E-05	2.55E-05	7.80E-11
15	3.769800197243	3.769800189662	7.58E-09	3.769843573640	3.769800189713	4.34E-05	4.34E-05	5.10E-11
16	4.669781984771	4.669781966348	1.84E-08	4.669852266020	4.669781966252	7.03E-05	7.03E-05	9.60E-11
17	5.730012761311	5.730012720038	4.13E-08	5.730121980482	5.730012720209	1.09E-04	1.09E-04	1.71E-10
18	6.969948947320	6.969948861216	8.61E-08	6.970112621145	6.969948861147	1.64E-04	1.64E-04	6.90E-11
19	8.410197286887	8.410197118139	1.69E-07	8.410434867735	8.410197118426	2.38E-04	2.38E-04	2.87E-10
20	10.072521332264	10.072521019159	3.13E-07	10.072856609111	10.072521019540	3.36E-04	3.35E-04	3.81E-10
21	11.979846181320	11.979845628123	5.53E-07	11.980307615185	11.979845628923	4.62E-04	4.61E-04	8.00E-10
22	14.156261974491	14.156261038555	9.36E-07	14.156882957145	14.156261039266	6.22E-04	6.21E-04	7.11E-10
23	16.627026501784	16.627024979348	1.52E-06	16.627845530788	16.627024981510	8.21E-04	8.19E-04	2.16E-09
24	19.418567164901	19.418564773503	2.39E-06	19.419627931210	19.418564777274	1.06E-03	1.06E-03	3.77E-09
25	22.558482468836	22.558478830451	3.64E-06	22.559833857208	22.558478838118	1.36E-03	1.35E-03	7.67E-09
26	26.075543162820	26.075537784858	5.38E-06	26.077239168646	26.075537799015	1.70E-03	1.70E-03	1.42E-08
27	29.999693124463	29.999685382422	7.74E-06	30.001792691151	29.999685407249	2.11E-03	2.10E-03	2.48E-08
28	34.362050046708	34.362039165277	1.09E-05	34.364616830678	34.362039204644	2.58E-03	2.57E-03	3.94E-08
29	39.194905974805	39.194891011872	1.50E-05	39.198008049120	39.194891079165	3.12E-03	3.10E-03	6.73E-08
30	44.531727731649	44.531707567688	2.02E-05	44.535437233170	44.531707674130	3.73E-03	3.71E-03	1.06E-07

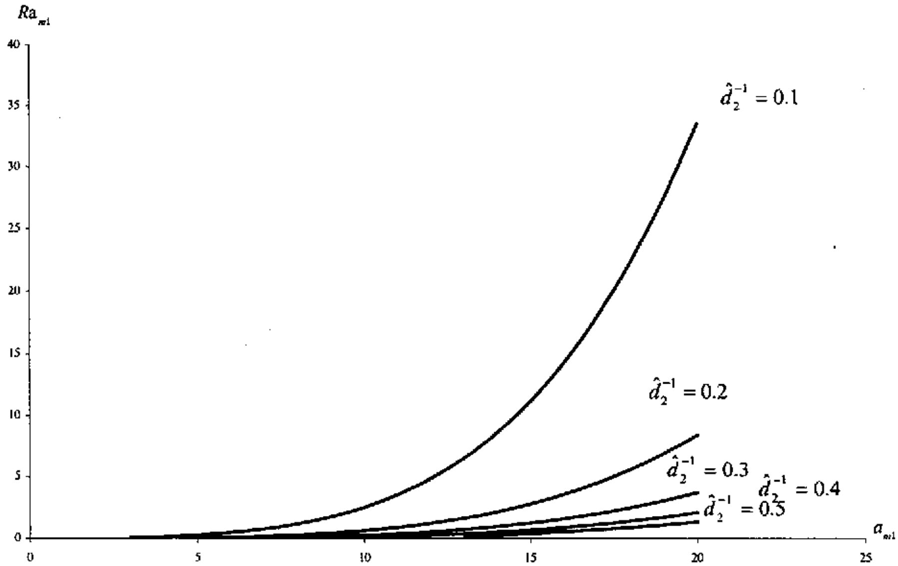

Fig. 2. The relation between a_{m1} and Ra_{m1} by using the first order of Chebyshev spectral method when $\hat{d}_1^{-1} = 1$, $M = 20$ and $M = 40$.

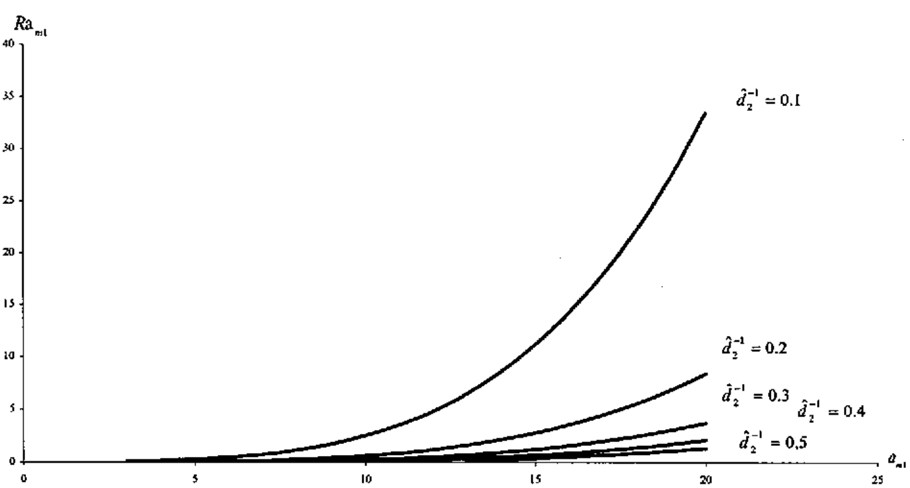

Fig. 3. The relation between a_{m1} and Ra_{m1} by using the second order of Chebyshev spectral method when $\hat{d}_1^{-1} = 1$, $M = 20$ and $M = 40$.

Table 5. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 1$ and $\hat{d}_2^{-1} = 0.5$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
13	0.083240910936	0.083240910944	8.00E-12	0.083240911960	0.083240910633	1.33E-09	1.02E-09	3.11E-10
14	0.238366977363	0.238366977354	9.00E-12	0.238366981538	0.238366977571	3.97E-09	4.18E-09	2.17E-10
15	0.383910238141	0.383910238042	9.90E-11	0.383910244286	0.383910238164	6.12E-09	6.15E-09	1.22E-10
16	0.530881684309	0.530881684127	1.82E-10	0.530881691213	0.530881684124	7.09E-09	6.90E-09	3.00E-12
17	0.691566087474	0.691566087109	3.65E-10	0.691566095185	0.691566087137	8.05E-09	7.71E-09	2.80E-11
18	0.874715126038	0.874715125476	5.62E-10	0.874715135276	0.874715125405	9.87E-09	9.24E-09	7.10E-11
19	1.086223438131	1.086223437309	8.22E-10	1.086223449973	1.086223437281	1.27E-08	1.18E-08	2.80E-11
20	1.330754062575	1.330754061452	1.12E-09	1.330754078569	1.330754061458	1.71E-08	1.60E-08	6.00E-12

Table 4. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 1$ and $\hat{d}_2^{-1} = 0.4$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
10	0.000000552253	0.000000552252	1.00E-12	0.000000552217	0.000000552314	9.70E-11	3.60E-11	6.20E-11
11	0.128812726274	0.128812726275	1.00E-12	0.128812726413	0.128812726297	1.16E-10	1.39E-10	2.20E-11
12	0.248571542351	0.248571542351	0.00E+00	0.248571542729	0.248571542348	3.81E-10	3.78E-10	3.00E-12
13	0.372619499288	0.372619499268	2.00E-11	0.372619500127	0.372619499265	8.62E-10	8.39E-10	3.00E-12
14	0.513353921159	0.513353921110	4.90E-11	0.513353923053	0.513353921138	1.91E-09	1.89E-09	2.80E-11
15	0.679363667089	0.679363667052	3.70E-11	0.679363671230	0.679363667009	4.22E-09	4.14E-09	4.30E-11
16	0.876991881098	0.876991881024	7.40E-11	0.876991889402	0.876991881026	8.38E-09	8.30E-09	2.00E-12
17	1.111759581834	1.111759581297	5.37E-10	1.111759597336	1.111759581448	1.59E-08	1.55E-08	1.51E-10
18	1.389001833968	1.389001833764	2.04E-10	1.389001862358	1.389001833923	2.84E-08	2.84E-08	1.59E-10
19	1.714127066463	1.714127066005	4.58E-10	1.714127114641	1.714127066198	4.84E-08	4.82E-08	1.93E-10
20	2.092727883978	2.092727883342	6.36E-10	2.092727962860	2.092727883581	7.93E-08	7.89E-08	2.39E-10

Table 3. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 1$ and $\hat{d}_2^{-1} = 0.3$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
8	0.049677378447	0.049677378447	0.00E+00	0.049677378453	0.049677378463	1.00E-11	6.00E-12	1.60E-11
9	0.144721479565	0.144721479565	0.00E+00	0.144721479710	0.144721479570	1.40E-10	1.45E-10	5.00E-12
10	0.247564815744	0.247564815742	2.00E-12	0.247564816640	0.247564815654	9.86E-10	8.96E-10	8.80E-11
11	0.370769390556	0.370769390545	1.10E-11	0.370769393878	0.370769390563	3.31E-09	3.32E-09	1.80E-11
12	0.523420629505	0.523420629499	6.00E-12	0.523420639004	0.523420629529	9.48E-09	9.50E-09	3.00E-11
13	0.713151147097	0.713151147082	1.50E-11	0.713151170146	0.713151147080	2.31E-08	2.30E-08	2.00E-12
14	0.947271914721	0.947271914699	2.20E-11	0.947271964557	0.947271914696	4.99E-08	4.98E-08	3.00E-12
15	1.233243367172	1.233243367083	8.90E-11	1.233243465910	1.233243367097	9.88E-08	9.87E-08	1.40E-11
16	1.578870669582	1.578870669407	1.75E-10	1.578870852140	1.578870669467	1.83E-07	1.83E-07	6.00E-11
17	1.992390083399	1.992390083065	3.34E-10	1.992390402264	1.992390083061	3.19E-07	3.19E-07	4.00E-12
18	2.482510131033	2.482510130671	3.62E-10	2.482510662107	2.482510130664	5.31E-07	5.31E-07	7.00E-12
19	3.058432620307	3.058432619717	5.90E-10	3.058433469148	3.058432619720	8.49E-07	8.49E-07	3.00E-12
20	3.729864048540	3.729864047660	8.80E-10	3.729865357728	3.729864047548	1.31E-06	1.31E-06	1.12E-10

Table 2. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 1$ and $\hat{d}_2^{-1} = 0.2$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
5	0.000823818206	0.000823818206	0.00E+00	0.000823818208	0.000823818213	5.00E-12	2.00E-12	7.00E-12
6	0.073982303122	0.073982303121	1.00E-12	0.073982303471	0.073982303130	3.41E-10	3.49E-10	9.00E-12
7	0.159707850671	0.159707850671	0.00E+00	0.159707853753	0.159707850676	3.08E-09	3.08E-09	5.00E-12
8	0.271604837102	0.271604837102	0.00E+00	0.271604852096	0.271604837095	1.50E-08	1.50E-08	7.00E-12
9	0.421151162806	0.421151162806	0.00E+00	0.421151215568	0.421151162795	5.28E-08	5.28E-08	1.10E-11
10	0.619594384742	0.619594384741	1.00E-12	0.619594534929	0.619594384744	1.50E-07	1.50E-07	3.00E-12
11	0.878739649871	0.878739649861	1.00E-11	0.878740016993	0.878739649878	3.67E-07	3.67E-07	1.70E-11
12	1.211277631791	1.211277631755	3.60E-11	1.211278431094	1.211277631741	7.99E-07	7.99E-07	1.40E-11
13	1.630928556647	1.630928556540	1.07E-10	1.630930144746	1.630928556641	1.59E-06	1.59E-06	1.01E-10
14	2.152509837958	2.152509837662	2.96E-10	2.152512767270	2.152509837649	2.93E-06	2.93E-06	1.30E-11
15	2.791969977888	2.791969976842	1.05E-09	2.791975057872	2.791969976844	5.08E-06	5.08E-06	2.00E-12
16	3.566406591318	3.566406588848	2.47E-09	3.566414955051	3.566406588841	8.37E-06	8.36E-06	7.00E-12
17	4.494076502321	4.494076496770	5.55E-09	4.494089674134	4.494076496946	1.32E-05	1.32E-05	1.76E-10
18	5.594401660859	5.594401649494	1.14E-08	5.594421624039	5.594401649452	2.00E-05	2.00E-05	4.20E-11
19	6.887972753687	6.887972731327	2.24E-08	6.888002013575	6.887972731336	2.93E-05	2.93E-05	9.00E-12
20	8.396551481884	8.396551442041	3.98E-08	8.396593125907	8.396551441991	4.17E-05	4.16E-05	5.00E-11

Table 1. Rayleigh number Ra_{m1} corresponding to wave number a_{m1} by using the first order and the second order of Chebyshev spectral methods when $\hat{d}_1^{-1} = 1$ and $\hat{d}_2^{-1} = 0.1$

a_{m1}	First order of Chebyshev spectral method			Second order of Chebyshev spectral method			Comparison between both methods	
	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Ra_{m1} when $M = 20$	Ra_{m1} when $M = 40$	Accuracy	Accuracy for $M = 20$	Accuracy for $M = 40$
3	0.054298017972	0.054298017972	0.00E+00	0.054298019980	0.054298017969	2.01E-09	2.01E-09	3.00E-12
4	0.139384864028	0.139384864029	1.00E-12	0.139384910402	0.139384864028	4.64E-08	4.64E-08	1.00E-12
5	0.267917908737	0.267917908733	4.00E-12	0.267918289531	0.267917908729	3.81E-07	3.81E-07	4.00E-12
6	0.463456055675	0.463456055605	7.00E-11	0.463457895320	0.463456055602	1.84E-06	1.84E-06	3.00E-12
7	0.752938082695	0.752938081987	7.08E-10	0.752944459246	0.752938081998	6.38E-06	6.38E-06	1.10E-11
8	1.167445496193	1.167445491588	4.61E-09	1.167463066504	1.167445491576	1.76E-05	1.76E-05	1.20E-11
9	1.742487236830	1.742487215305	2.15E-08	1.742528155279	1.742487215360	4.09E-05	4.09E-05	5.50E-11
10	2.518130333055	2.518130254792	7.83E-08	2.518214152269	2.518130254930	8.39E-05	8.38E-05	1.38E-10
11	3.539065493626	3.539065259639	2.34E-07	3.539220739282	3.539065259831	1.55E-04	1.55E-04	1.92E-10
12	4.854641791264	4.854641193408	5.98E-07	4.854906982800	4.854641194457	2.66E-04	2.65E-04	1.05E-09
13	6.518885790694	6.518884446233	1.34E-06	6.519309792161	6.518884449588	4.25E-04	4.24E-04	3.35E-09
14	8.590512511886	8.590509791513	2.72E-06	8.591154207683	8.590509801659	6.44E-04	6.42E-04	1.01E-08
15	11.132931932549	11.132926891884	5.04E-06	11.133859308292	11.132926918645	9.32E-04	9.27E-04	2.68E-08
16	14.214252959803	14.214244289031	8.67E-06	14.215541714459	14.214244350965	1.30E-03	1.29E-03	6.19E-08
17	17.907285898152	17.907271897305	1.40E-05	17.909017743381	17.907272031099	1.75E-03	1.73E-03	1.34E-07
18	22.289544000267	22.289522592089	2.14E-05	22.291804795247	22.289522860199	2.28E-03	2.26E-03	2.68E-07
19	27.443244472593	27.443213210634	3.13E-05	27.446122288476	27.443213721547	2.91E-03	2.88E-03	5.11E-07
20	33.455308979936	33.455265185885	4.38E-05	33.458892323056	33.455266098326	3.63E-03	3.58E-03	9.12E-07

$\hat{d}_1^{-1} = 0.5$ respectively. On other hand from the tables we observe that the accuracy of first order of Chebyshev spectral method is more accurate than the accuracy of second order of Chebyshev spectral method. For example, in Table 1 at $\hat{d}_1^{-1} = 1, \hat{d}_2^{-1} = 0.1$ and $a_{m1} = 4$, it found that the accuracy of first order of Chebyshev spectral method is $1.00\mathrm{E}-12$ whereas the accuracy of second order of Chebyshev spectral method is $4.64\mathrm{E}-08$. Also it noticeable that in both methods as M increase the accuracy increase. This disparity in precision results of both methods dose not becomes visible in figures for the reason that the dissimilarity is very small. So we conclude that the first order of Chebyshev spectral method as we believe is better than the second order of Chebyshev spectral method. Therefore we advise for researchers to use the first order of Chebyshev spectral method on solving the eigenvalue problems of linear stability.

ordinary differential equations of first order in fluid layer and system of four ordinary differential equations of first order in every porous layer with 14 boundary conditions while by way of using second order of Chebyshev spectral method, it transform to system of three ordinary differential equations of second order in fluid layer and system of two ordinary differential equations of second order in every porous layer with 14 boundary conditions. In both methods, the final form of eigenvalue problem handle with NAG routine F02BJF where the matrices E and F are real. Numerical results by using both methods are display in Tables 1, 2, 3, 4 and 5 for $\hat{d}_1^{-1}=1$ and Tables 6, 7, 8, 9 and 10 for $\hat{d}_1^{-1}=0.5$ and variant values of reciprocal depth ratio $\hat{d}_2^{-1}$, and different values of number of Chebyshev polynomials M where Darcy number $Da_2=4\times10^{-6}$, thermal conductivity ratio $\hat{k}_1^{-1}=\hat{k}_2^{-1}=0.7$, Beavers-Joseph constant $\alpha_{BJ}=0.1$, porosity $\phi=1$, Prandtl number of the fluid layer $P_{r_f}=1$ and $G_{m1}=G_{m2}=1$.

By going through these tables we observe that as $\hat{d}_2^{-1}$ increase Ra_{m1} decrease. This illustrate in Figs. 2 and 4 which show the relation between wave number a_{m1} and Rayleigh number Ra_{m1} by using the first order of Chebyshev spectral method when $\hat{d}_1^{-1}=1$ and $\hat{d}_1^{-1}=0.5$ respectively whereas Figs. 3 and 5 show the relation between wave number a_{m1} and Rayleigh number Ra_{m1} by using the second order of Chebyshev spectral method when $\hat{d}_1^{-1}=1$ and

where A and B are real 7×7 matrices. The final eigenvalue problem reduce to $EV=\sigma FV$ where matrices E and F have block form .The boundary conditions replace the $(M-1)$th, Mth, $(2M-1)$th, $2M$th, …, $(7M-1)$th and $7M$th row of E and F

$$y_1=0, \qquad y_2=0, \qquad \text{on } z=1$$

$$\varepsilon_{T1}y_3(1)-y_1(-1)=0, \qquad y_5(1)-\varepsilon_{T1}y_2(-1)=0, \qquad Dy_5(1)-\hat{d}_1Dy_2(-1)=0,$$

$$\varepsilon_{T1}\hat{d}_1^3Da_1\left(Dy_4(1)-3a_f^2y_3(1)\right)+\hat{d}_1Dy_1(-1)=\sigma\left(\varepsilon_{T1}\hat{d}_1^3\frac{\hat{k}_1Da_1}{\hat{d}_1^2P_{r_f}}Dy_3(1)-\frac{Da_1}{P_{r_{m1}}\phi}\hat{d}_1Dy_1(-1)\right),$$

$$\varepsilon_{T1}\hat{d}_1\left(Dy_3(1)-\frac{\hat{d}_1\sqrt{Da_1}}{\alpha_{BJ}}y_4(1)\right)-\hat{d}_1Dy_1(-1)=0,$$

$$\varepsilon_{T2}y_3(-1)-y_6(1)=0, \qquad y_5(-1)-\varepsilon_{T2}y_7(1)=0, \qquad Dy_5(-1)-Dy_7(1)=0,$$

$$\varepsilon_{T2}\hat{d}_2^3Da_2\left(Dy_4(-1)-3a_f^2Dy_3(-1)\right)+Dy_6(1)=\sigma\left(\varepsilon_{T2}\hat{d}_2^3\frac{\hat{k}_1Da_2}{\hat{d}_1^2P_{r_f}}Dy_3(-1)-\frac{\hat{d}_2^2\hat{k}_1Da_2}{\hat{d}_1^2\hat{k}_2P_{r_{m2}}\phi}Dy_6(1)\right),$$

$$\varepsilon_{T2}\hat{d}_2\left(Dy_3(-1)-\frac{\hat{d}_2\sqrt{Da_2}}{\alpha_{BJ}}y_4(-1)\right)-Dy_6(1)=0,$$

$$y_6=0, \qquad y_7=0. \qquad \text{on } z=-1$$

(4.3)

Numerical Results And Conclusions

To compare Chebyshev spectral methods of the first order and the second order we solve eigenvalue problem (2.24) with boundary conditions (2.25), (2.26) and (2.27) by using both methods. By using first order of Chebyshev spectral method, the eigenvalue problem transform to system of six

be Chebyshev spectral expansion up to order M, and let the variables y_r where $1 \le r \le 7$ be defined by

$$y_1 = w_{m1}, \qquad y_2 = \theta_{m1}, \qquad y_3 = w_f, \qquad y_4 = D_f^2 w_f, \qquad y_5 = \theta_f,$$
$$y_6 = w_{m2}, \qquad y_7 = \theta_{m2}. \tag{4.1}$$

thence the basic equations (2.24) can be rewritten in system of seven ordinary differential equations of second order as follows

$$D_{m1}^2 y_1 = a_{m1}^2 y_1 - a_{m1}^2 Ra_{m1} y_2 - \sigma_{m1} \frac{Da_1}{P_{r_{m1}}\phi}\left(D_{m1}^2 - a_{m1}^2\right) y_1,$$

$$D_{m1}^2 y_2 = a_{m1}^2 y_2 - y_1 + \sigma_{m1} G_{m1} y_2,$$

$$D_f^2 y_3 = y_4,$$

$$D_f^2 y_4 = 2a_f^2 y_4 - a_f^4 y_3 + a_f^2 Ra_f y_5 + \frac{\sigma_f}{P_{r_f}}\left(y_4 - a_f^2 y_3\right), \tag{4.2}$$

$$D_f^2 y_5 = a_f^2 y_5 - y_3 + \sigma_f y_5,$$

$$D_{m2}^2 y_6 = a_{m2}^2 y_6 - a_{m2}^2 Ra_{m2} y_7 - \sigma_{m2} \frac{Da_2}{P_{r_{m2}}\phi}\left(D_{m2}^2 - a_{m2}^2\right) y_6,$$

$$D_{m2}^2 y_7 = a_{m2}^2 y_7 - y_6 + \sigma_{m2} G_{m2} y_7.$$

Since $D_{m1} = \hat{d}_1 D$, $D_f = D_{m2} = D$ and if we put $\sigma_{m1} = \sigma$ then $\sigma_f = \frac{\hat{k}_1}{\hat{d}_1^2}\sigma, \sigma_{m2} = \frac{\hat{d}_2^2 \hat{k}_1}{\hat{d}_1^2 \hat{k}_2}\sigma$ so the eigenvalue problem can be reformed it in the form

$$\frac{d^2 Y}{dz^2} = AY + \sigma BY, \qquad z \in [-1,1]$$

$$\frac{dY}{dz} = AY + \sigma BY, \qquad z \in [-1,1]$$

where A and B are real 14×14 matrices. The final eigenvalue problem reduce to $EV = \sigma FV$ where matrices E and F have block form .The boundary conditions replace the Mth, $2M$th, $3M$th, …,$14M$th row of E and F

$$y_1 = 0, \qquad y_3 = 0, \qquad \text{on}\, z = 1$$

$$\varepsilon_{T1} y_5(1) - y_1(-1) = 0, \qquad y_9(1) - \varepsilon_{T1} y_3(-1) = 0, \qquad y_{10}(1) - y_4(-1) = 0,$$

$$\varepsilon_{T1}\hat{d}_1^3 Da_1\left(y_8(1) - 3a_f^2 y_6(1)\right) + y_2(-1) = \sigma\left(\varepsilon_{T1}\hat{d}_1^3 \frac{\hat{k}_1 Da_1}{\hat{d}_1^2 P_{r_f}} y_6(1) - \frac{Da_1}{P_{r_{m1}}\phi} y_2(-1)\right),$$

$$\varepsilon_{T1}\hat{d}_1\left(y_6(1) - \frac{\hat{d}_1\sqrt{Da_1}}{\alpha_{BJ}} y_7(1)\right) - y_2(-1) = 0,$$

$$\varepsilon_{T2} y_5(-1) - y_{11}(1) = 0, \qquad y_9(-1) - \varepsilon_{T2} y_{13}(1) = 0, \qquad y_{10}(-1) - y_{14}(1) = 0,$$

$$\varepsilon_{T2}\hat{d}_2^3 Da_2\left(y_8(-1) - 3a_f^2 y_6(-1)\right) + y_{12}(1) = \sigma\left(\varepsilon_{T2}\hat{d}_2^3 \frac{\hat{k}_1 Da_2}{\hat{d}_1^2 P_{r_f}} y_6(-1) - \frac{\hat{d}_2^2\hat{k}_1 Da_2}{\hat{d}_1^2\hat{k}_2 P_{r_{m2}}\phi} y_{12}(1)\right),$$

$$\varepsilon_{T2}\hat{d}_2\left(y_6(-1) - \frac{\hat{d}_2\sqrt{Da_2}}{\alpha_{BJ}} y_7(-1)\right) - y_{12}(1) = 0,$$

$$y_{11} = 0, \qquad y_{13} = 0. \qquad \text{on}\, z = -1$$

(3.3)

The Solution By Second Order Of Chebyshev Spectral Method

Suppose that

$$y_r(z) = \sum_{k=0}^{M-1} \alpha_{kr} T_k(z), \qquad 1 \le r \le 7, \quad z \in [-1,1]$$

$$D_{m1} y_1 = y_2,$$

$$D_{m1} y_2 = a_{m1}^2 y_1 - a_{m1}^2 Ra_{m1} y_3 + \sigma_{m1} \frac{Da_1}{P_{r_{m1}} \phi}\left(a_{m1}^2 y_1 - D_{m1} y_2\right),$$

$$D_{m1} y_3 = y_4,$$

$$D_{m1} y_4 = a_{m1}^2 y_3 - y_1 + \sigma_{m1} G_{m1} y_3,$$

$$D_f y_5 = y_6,$$

$$D_f y_6 = y_7,$$

$$D_f y_7 = y_8,$$

$$D_f y_8 = 2a_f^2 y_7 - a_f^4 y_5 + a_f^2 Ra_f y_9 + \frac{\sigma_f}{P_{r_f}}\left(y_7 - a_f^2 y_5\right),$$

$$D_f y_9 = y_{10},$$

$$D_f y_{10} = a_f^2 y_9 - y_5 + \sigma_f y_9,$$

$$D_{m2} y_{11} = y_{12},$$

$$D_{m2} y_{12} = a_{m2}^2 y_{11} - a_{m2}^2 Ra_{m2} y_{13} + \sigma_{m2} \frac{Da_2}{P_{r_{m2}} \phi}\left(a_{m2}^2 y_{11} - D_{m2} y_{12}\right), \qquad (3.2)$$

$$D_{m2} y_{13} = y_{14},$$

$$D_{m2} y_{14} = a_{m2}^2 y_{13} - y_{11} + \sigma_{m2} G_{m2} y_{13}.$$

Since $D_{m1} = \hat{d}_1 D$, $D_f = D_{m2} = D$ and if we put $\sigma_{m1} = \sigma$ then $\sigma_f = \frac{\hat{k}_1}{\hat{d}_1^2}\sigma, \sigma_{m2} = \frac{\hat{d}_2^2 \hat{k}_1}{\hat{d}_1^2 \hat{k}_2}\sigma$ so the eigenvalue problem can be reformed it in the form

$$w_{m1}=0, \qquad \theta_{m1}=0, \qquad \text{on } x_3=\frac{d_{m1}}{\left(d_{m1}-d_f\right)} \tag{2.25}$$

$$\left.\begin{array}{l} \varepsilon_{Ti}w_f=w_{mi}, \quad \theta_f=\varepsilon_{Ti}\theta_{mi}, \quad D_f\theta_f=D_{mi}\theta_{mi}, \\ \hat{d}_i^3\varepsilon_{Ti}Da_i\left(D_f^3w_f-3\alpha_f^2D_fw_f-\dfrac{\sigma_f}{P_{r_f}}D_fw_f\right)=-\left(\dfrac{Da_i}{P_{r_{mi}}\phi}\sigma_{mi}+1\right)D_{mi}w_{mi}, \\ \varepsilon_{Ti}\hat{d}_i\left(D_fw_f-\dfrac{\hat{d}_i\sqrt{Da_i}}{\alpha_{BJ}}D_f^2w_f\right)=D_{mi}w_{mi}, \end{array}\right\} \begin{array}{l} \text{on} x_3=1 \text{ at } i=1 \\ \text{on} x_3=0 \text{ at } i=2 \end{array} \tag{2.26}$$

$$w_{m2}=0, \qquad \theta_{m2}=0. \qquad \text{on } x_3=-1 \tag{2.27}$$

The Solution By First Order Of Chebyshev Spectral Method

Suppose that

$$y_r(z)=\sum_{k=0}^{M-1}\alpha_{kr}T_k(z), \qquad 1\le r\le 14, \quad z\in[-1,1]$$

be Chebyshev spectral expansion up to order M, and let the variables y_r where $1\le r\le 14$ be defined by

$$\begin{array}{llll} y_1=w_{m1}, & y_2=D_{m1}w_{m1}, & y_3=\theta_{m1}, & y_4=D_{m1}\theta_{m1}, \\ y_5=w_f, & y_6=D_fw_f, & y_7=D_f^2w_f, & y_8=D_f^3w_f, \\ y_9=\theta_f, & y_{10}=D_f\theta_f, & & \\ y_{11}=w_{m2}, & y_{12}=D_{m2}w_{m2}, & y_{13}=\theta_{m2}, & y_{14}=D_{m2}\theta_{m2}. \end{array} \tag{3.1}$$

thence the basic equations (2.24) can be rewritten in system of fourteen ordinary differential equations of first order as follows

$$
\begin{aligned}
\frac{\sigma_f}{P_{r_f}}\left(D_f^2 - a_f^2\right) w_f &= \left(D_f^2 - a_f^2\right)^2 w_f - \mathrm{Ra}_f a_f^2 \theta_f, \\
\sigma_f \theta_f &= w_f + \left(D_f^2 - a_f^2\right) \theta_f, \\
-\frac{Da}{\phi}\frac{\sigma_{mi}}{P_{r_{mi}}}\left(D_{mi}^2 - a_{mi}^2\right) w_{mi} &= \left(D_{mi}^2 - a_{mi}^2\right) w_{mi} + \mathrm{Ra}_{mi} a_{mi}^2 \theta_{mi}, \\
G_{mi}\sigma_{mi}\theta_{mi} &= w_{mi} + \left(D_{mi}^2 - a_{mi}^2\right) \theta_{mi}.
\end{aligned} \tag{2.24}
$$

where $a_f = \sqrt{l_f^2 + n_f^2}$ and $a_{mi} = \sqrt{l_{mi}^2 + n_{mi}^2}$ are non-dimensionalised wave number in the fluid layer and porous medium respectively, σ_f and σ_{mi} are non- dimensionalised constant time for the fluid and porous medium respectively and

$$
a_f = \frac{a_{m1}}{\hat{d}_1}, \qquad a_{m2} = \frac{\hat{d}_2 a_{m1}}{\hat{d}_1},
$$

$$
\sigma_f = \frac{\hat{k}_1}{\hat{d}_1^2}\sigma_{m1}, \qquad \sigma_{m2} = \frac{\hat{d}_2^2 \hat{k}_1}{\hat{d}_1^2 \hat{k}_2}\sigma_{m1},
$$

$$
D_{m1} = \frac{\partial}{\partial x_3} \qquad 1 < x_3 < \frac{d_{m1}}{\left(d_{m1} - d_f\right)},
$$

$$
D_f = \frac{\partial}{\partial x_3} \qquad 0 < x_3 < 1,
$$

$$
D_{m2} = \frac{\partial}{\partial x_3} \qquad -1 < x_3 < 0.
$$

The boundary conditions in the final form are

$$P_{r_{m1}} = \frac{\mu}{\rho_0 \lambda_{m1}}, \qquad Ra_{m1} = \frac{g\rho_0 \alpha K(d_{m1} - d_f)|T_1 - T_u|}{\mu\ \lambda_{m1}}, \qquad Da_1 = \frac{K}{(d_{m1} - d_f)^2}.$$

$$P_{r_f} = \frac{\mu}{\rho_0 \lambda_f}, \qquad Ra_f = \frac{g\rho_0 \alpha\, d_f^3 |T_0 - T_1|}{\mu\ \lambda_f},$$

$$P_{r_{m2}} = \frac{\mu}{\rho_0 \lambda_{m2}}, \qquad Ra_{m2} = \frac{g\rho_0 \alpha\, K d_{m2} |T_l - T_0|}{\mu\ \lambda_{m2}}, \qquad Da_2 = \frac{K}{d_{m2}^2}.$$

Linearisation will be done by neglect all products and powers (higher than the first) of the linear perturbation quantity, and by drop the (^) superscript, then we take double curl of the first equation in each layer to eliminate the hydrostatic pressure and we take the third component of the reworked momentum equation in each layer.

The normal stress and the Beaver-Joseph boundary conditions on the interface plane become

$$\hat{d}_i^3 \varepsilon_{Ti} Da_i \frac{\partial}{\partial x_3}\left(\nabla^2 w_f - \frac{1}{P_{r_f}}\frac{\partial w_f}{\partial t} + 2\Delta_2 w_f\right) = -\left(\frac{Da_i}{P_{r_{mi}}\phi}\frac{\partial}{\partial t} + 1\right)\frac{\partial w_{mi}}{\partial x_3} \tag{2.21}$$

$$\varepsilon_{Ti}\hat{d}_i \frac{\partial}{\partial x_3}\left(w_f - \frac{\hat{d}_i\sqrt{Da_i}}{\alpha_{BJ}}\frac{\partial w_f}{\partial x_3}\right) = \frac{\partial w_{mi}}{\partial x_3} \qquad , i = 1, 2 \tag{2.22}$$

where (2.21) obtained from $(2.20)_4$ by using two-dimensional Laplacian and divergence properties whereas (2.22) obtained from $(2.20)_5$ and $(2.20)_6$ by using two-dimensional divergence of the tangential components of the fluid velocity.

We apply the normal modes solution of the form

$$\psi(x,t) = \psi(x_3)\exp\left[i(lx_1 + nx_2) + \sigma t\right] \tag{2.23}$$

where ψ denote to the variables w_f, θ_f, w_{mi} and θ_{mi}, $i = 1,2$, on the consequence equations to get

and the governing equations of the lower porous medium can be written in the non-dimensional form

$$\begin{aligned}
&\frac{Da_2}{\phi}\frac{\partial \hat{\boldsymbol{v}}_{m2}}{\partial \hat{t}_{m2}} = P_{r_{m2}}\left(-\nabla_{m2}\hat{p}_{m2} - \hat{\boldsymbol{v}}_{m2} + Ra_{m2}\hat{\theta}_{m2}\boldsymbol{e}_3\right), \\
&G_{m2}\frac{\partial \hat{\theta}_{m2}}{\partial \hat{t}_{m2}} + \hat{\boldsymbol{v}}_{m2}\cdot\left(\nabla_{m2}\hat{\theta}_{m2} - sign(T_l - T_0)\boldsymbol{e}_3\right) = \nabla^2_{m2}\hat{\theta}_{m2}.
\end{aligned} \tag{2.19}$$

By using (2.14), (2.15) and (2.16) the boundary conditions become

$$\hat{w}_{m1} = 0, \qquad \hat{\theta}_{m1} = 0. \qquad \text{on } x_3 = \frac{d_{m1}}{\left(d_{m1} - d_f\right)}$$

$$\left.\begin{aligned}
&\varepsilon_{Ti}\hat{w}_f = \hat{w}_{mi}, \quad \hat{\theta}_f = \varepsilon_{Ti}\hat{\theta}_{mi}, \quad \frac{\partial \hat{\theta}_f}{\partial \hat{x}_3} = \frac{\partial \hat{\theta}_{mi}}{\partial \hat{x}_3}, \\
&\varepsilon_{Ti}\hat{d}_i Da_i\left(\hat{p}_f - 2\frac{\partial \hat{w}_f}{\partial \hat{x}_3}\right) = \hat{p}_{mi}, \\
&\varepsilon_{Ti}\frac{\partial \hat{u}_f}{\partial \hat{x}_3} = \frac{\alpha_{BJ}}{\hat{d}_i\sqrt{Da_i}}\left(\varepsilon_{Ti}\hat{u}_f - \hat{u}_{mi}\right), \\
&\varepsilon_{Ti}\frac{\partial \hat{v}_f}{\partial \hat{x}_3} = \frac{\alpha_{BJ}}{\hat{d}_i\sqrt{Da_i}}\left(\varepsilon_{Ti}\hat{v}_f - \hat{v}_{mi}\right),
\end{aligned}\right\} \begin{aligned}&\text{on } x_3 = 1 \text{ at } i = 1\\ &\text{on } x_3 = 0 \text{ at } i = 2\end{aligned} \tag{2.20}$$

$$\hat{w}_{m2} = 0, \qquad \hat{\theta}_{m2} = 0. \qquad \text{on } x_3 = -1$$

We define $G_{mi} = (\rho c)_{mi}/(\rho c_p)_f$ and since the porous medium is heat from below then

$$sign(T_l - T_0) = sign(T_0 - T_1) = sign(T_1 - T_u) = 1$$

The non-dimensional numbers $P_{r_{m1}}, Ra_{m1}$ and Da_1 are symbolize to the Prandtl number, Rayleigh number and Darcy number of the upper porous layer whereas P_{r_f} and Ra_f are symbolize to the Prandtl number and Rayleigh number of the fluid layer and $P_{r_{m2}}, Ra_{m2}$ and Da_2 are symbolize to the Prandtl number, Rayleigh number and Darcy number of the lower porous layer. These numbers are given by

$$x = d_f \hat{x}_f, \qquad t_f = \frac{d_f^2}{\lambda_f}\hat{t}_f, \qquad v_f = \frac{\lambda_f}{d_f}\hat{v}_f,$$

$$p_f = \frac{\mu\lambda_f}{d_f^2}\hat{p}_f, \qquad \theta_f = |T_0 - T_1|\hat{\theta}_f. \tag{2.15}$$

for the fluid layer L_2, and by using

$$x = d_{m2}\hat{x}_{m2}, \qquad t_{m2} = \frac{d_{m2}^2}{\lambda_{m2}}\hat{t}_{m2}, \qquad v_{m2} = \frac{\lambda_{m2}}{d_{m2}}\hat{v}_{m2},$$

$$p_{m2} = \frac{\mu\lambda_{m2}}{K}\hat{p}_{m2}, \qquad \theta_{m2} = |T_l - T_0|\hat{\theta}_{m2}. \tag{2.16}$$

for the second porous medium L_3, where $\lambda_f = k_f/(\rho c_p)_f$ and $\lambda_m = k_m/(\rho c_p)_f$ are thermal diffusivity of the fluid phase and porous mediums respectively and putting

$$\hat{d}_1 = (d_{m1} - d_f)/d_f, \qquad \hat{k}_1 = k_{m1}/k_f, \qquad \varepsilon_{T1} = \hat{d}_1/\hat{k}_1,$$

$$\hat{d}_2 = d_{m2}/d_f, \qquad \hat{k}_2 = k_{m2}/k_f, \qquad \varepsilon_{T2} = \hat{d}_2/\hat{k}_2.$$

Hence the governing equations of the upper porous medium can be written in the non-dimensional form

$$\frac{Da_1}{\phi}\frac{\partial\hat{v}_{m1}}{\partial\hat{t}_{m1}} = P_{r_{m1}}\left(-\nabla_{m1}\hat{p}_{m1} - \hat{v}_{m1} + Ra_{m1}\hat{\theta}_{m1}e_3\right),$$

$$G_{m1}\frac{\partial\hat{\theta}_{m1}}{\partial\hat{t}_{m1}} + \hat{v}_{m1}\cdot\left(\nabla_{m1}\hat{\theta}_{m1} - sign(T_1 - T_u)e_3\right) = \nabla_{m1}^2\hat{\theta}_{m1}. \tag{2.17}$$

whereas the governing equations of the fluid layer can be written in the non-dimensional form

$$\frac{\partial\hat{v}_f}{\partial\hat{t}_f} + (\hat{v}_f\cdot\nabla_f)\hat{v}_f = P_{r_f}\left(-\nabla_f\hat{p}_f + \nabla_f^2\hat{v}_f + Ra_f\hat{\theta}_f e_3\right),$$

$$\frac{\partial\hat{\theta}_f}{\partial\hat{t}_f} + \hat{v}_f\cdot\left(\nabla_f\hat{\theta}_f - sign(T_0 - T_1)e_3\right) = \nabla_f^2\hat{\theta}_f. \tag{2.18}$$

$$T_f(d_f)=T_{m1}(d_f),\qquad k_f\frac{\partial T_f(d_f)}{\partial x_3}=k_{m1}\frac{\partial T_{m1}(d_f)}{\partial x_3},\qquad P_f(d_f)=P_{m1}(d_f),$$
$$T_f(0)=T_{m2}(0),\qquad k_f\frac{\partial T_f(0)}{\partial x_3}=k_{m2}\frac{\partial T_{m2}(0)}{\partial x_3},\qquad P_f(0)=P_{m2}(0). \tag{2.11}$$

In conclusion, it follows that the equilibrium temperature fields in the layers L_1, L_2 and L_3 respectively are

$$T_{m1}=\frac{(T_1-T_u)x_3+d_fT_u-d_{m1}T_1}{d_f-d_{m1}},\qquad d_f\le x_3\le d_{m1}$$
$$T_f=T_0-(T_0-T_1)\frac{x_3}{d_f},\qquad 0\le x_3\le d_f \tag{2.12}$$
$$T_{m2}=T_0-(T_l-T_0)\frac{x_3}{d_{m2}}.\qquad -d_{m2}\le x_3\le 0$$

We apply the perturbation by the following linear perturbation quantities

$$\mathbf{V}_{m1}=\mathbf{0}+\mathbf{v}_{m1},\qquad P_{m1}=P_{m1}(x_3)+p_{m1},\qquad T_{m1}=\frac{(T_1-T_u)x_3+d_fT_u-d_{m1}T_1}{d_f-d_{m1}}+\theta_{m1},$$
$$\mathbf{V}_f=\mathbf{0}+\mathbf{v}_f,\qquad P_f=P_f(x_3)+p_f,\qquad T_f=T_0-(T_0-T_1)\frac{x_3}{d_f}+\theta_f, \tag{2.13}$$
$$\mathbf{V}_{m2}=\mathbf{0}+\mathbf{v}_{m2},\qquad P_{m2}=P_{m2}(x_3)+p_{m2},\qquad T_{m2}=T_0-(T_l-T_0)\frac{x_3}{d_{m2}}+\theta_{m2},$$

to the governing equations (2.2) in the fluid layer and (2.3) in porous medium and to the boundary conditions.
After perturbation, the non-dimensionlisation will be apply by using

$$\mathbf{x}=(d_{m1}-d_f)\hat{\mathbf{x}}_{m1},\qquad t_{m1}=\frac{(d_{m1}-d_f)^2}{\lambda_{m1}}\hat{t}_{m1},\qquad \mathbf{v}_{m1}=\frac{\lambda_{m1}}{(d_{m1}-d_f)}\hat{\mathbf{v}}_{m1},$$
$$p_{m1}=\frac{\mu\lambda_{m1}}{K}\hat{p}_{m1},\qquad \theta_{m1}=|T_1-T_u|\hat{\theta}_{m1}. \tag{2.14}$$

for the first porous medium L_1, and by using

$$w_f(d_f)=w_{m1}(d_f), \qquad T_f(d_f)=T_{m1}(d_f),$$
$$k_f\frac{\partial T_f(d_f)}{\partial x_3}=k_{m1}\frac{\partial T_{m1}(d_f)}{\partial x_3}, \qquad -P_f(d_f)+2\mu\frac{\partial w_f(d_f)}{\partial x_3}=-P_{m1}(d_f). \tag{2.6}$$

and the boundary conditions on the interface plane $x_3=0$ are

$$w_f(0)=w_{m2}(0), \qquad T_f(0)=T_{m2}(0),$$
$$k_f\frac{\partial T_f(0)}{\partial x_3}=k_{m2}\frac{\partial T_{m2}(0)}{\partial x_3}, \qquad -P_f(0)+2\mu\frac{\partial w_f(0)}{\partial x_3}=-P_{m2}(0). \tag{2.7}$$

The interfacial conditions mean respectively the normal fluid velocity, temperature, heat flux, and normal stress are continuous. The final two conditions to be specified on $x_3=d_f$ and $x_3=0$ that suggested by Beavers-Joseph (1967) has the form

$$\frac{\partial u_f}{\partial x_3}=\frac{\alpha_{BJ}}{\sqrt{K}}\left(u_f-u_{mi}\right), \qquad \frac{\partial v_f}{\partial x_3}=\frac{\alpha_{BJ}}{\sqrt{K}}\left(v_f-v_{mi}\right), \quad ,i=1,2 \tag{2.8}$$

where $u_{m1},v_{m1},u_f,v_f,u_{m2}$ and v_{m2} are the limiting tangential components of the fluid velocity as the interface is approached from the layers L_1, L_2 and L_3 respectively and α_{BJ} is Beavers-Joseph constant.

Equations (2.2) and (2.3) have an equilibrium solution satisfying the boundary conditions (2.4)-(2.8) of the form

$$\mathbf{V}_f=\mathbf{0}, \qquad \mathbf{V}_{mi}=\mathbf{0},$$
$$-\nabla P_f+\rho_f\mathbf{g}=\mathbf{0}, \qquad -\nabla P_{mi}+\rho_f\mathbf{g}=\mathbf{0}, \qquad \nabla^2 T_f=\nabla^2 T_{mi}=0. \quad ,i=1,2 \tag{2.9}$$

together with the exterior boundary conditions

$$T_{m1}(d_{m1})=T_u, \qquad T_f(d_f)=T_1, \qquad T_{m2}(-d_{m2})=T_l. \tag{2.10}$$

and the interfacial conditions

solenoidal fluid velocity vector, P_f is the hydrostatic pressure, k_f is the thermal conductivity of the fluid, μ is the dynamic viscosity of the fluid and $(\rho c_p)_f$ is the heat capacity per unit volume of the fluid at constant pressure .

The governing equations of the porous medium are

$$\begin{aligned}&\frac{\rho_0}{\phi}\frac{\partial \boldsymbol{V}_{mi}}{\partial t}=-\nabla P_{mi}-\frac{\mu}{K}\boldsymbol{V}_{mi}+\rho_f\boldsymbol{g},\\&(\rho c)_{mi}\frac{\partial T_{mi}}{\partial t}+(\rho c_p)_f\boldsymbol{V}_{mi}\cdot\nabla T_{mi}=k_{mi}\nabla^2 T_{mi}.\end{aligned}\qquad ,i=1,2 \qquad (2.3)$$

where T_{mi} is the Kelvin temperature of the porous medium, $\boldsymbol{V}_{mi}$ is the solenoidal seepage velocity vector, P_{mi} is the hydrostatic pressure, K is the permeability of the porous substrate, ϕ is its porosity, k_{mi} is the overall thermal conductivity of the porous medium and $(\rho c)_{mi}$ is the overall heat capacity per unit volume of the porous medium at constant pressure which is given by

$$(\rho c)_{mi}=\phi(\rho c_p)_f+(1-\phi)(\rho c_p)_{mi}$$

where $(\rho c_p)_{mi}$ is the heat capacity per unit volume of the porous substrate.

The boundary conditions at $x_3=d_{m1}$ are

$$w_{m1}(d_{m1})=0, \qquad T_{m1}(d_{m1})=T_u, \qquad (2.4)$$

and the boundary conditions at $x_3=-d_{m2}$ are

$$w_{m2}(-d_{m2})=0, \qquad T_{m2}(-d_{m2})=T_l, \qquad (2.5)$$

where w_{m1} and w_{m2} are the axial velocity components of the fluid in L_1 and L_3 respectively and T_u and T_l are the temperature of the upper boundary of L_1 and the lower boundary of L_3 respectively.

The boundary conditions on the interface plane $x_3=d_f$ are

Suppose that the middle layer L_2 is filled by an incompressible viscous fluid which flow in it governed by Navier-Stokes equations, whereas the upper layer L_1 and the lower layer L_3 are occupied by a porous medium permeated by the fluid which flow in them governed by Darcy's law. Gravity $\boldsymbol{g}$ acts in the negative direction of x_3 and the lower porous medium is heated from below. The motion (convection) is driven by thermal buoyancy and damped by the viscosity. The Oberbeck-Boussinesq approximation is used so that the density is constant everywhere except in the body force term.

Let T is the Kelvin temperature of the fluid, T_0 is a constant reference Kelvin temperature, ρ_0 is the density of the fluid at T_0 and α is the coefficient of volume expansion of the fluid (constant), then the fluid density ρ_f is proportionate to T such that

$$\rho_f = \rho_0[1-\alpha(T-T_0)] \tag{2.1}$$

Mathematical Treatment

The governing equations of the fluid layer are

$$\begin{aligned}
&\rho_0\left(\frac{\partial \boldsymbol{V}_f}{\partial t}+(\boldsymbol{V}_f\cdot\nabla)\boldsymbol{V}_f\right)=-\nabla P_f+\mu\nabla^2\boldsymbol{V}_f+\rho_f\boldsymbol{g},\\
&(\rho c_p)_f\left(\frac{\partial T_f}{\partial t}+\boldsymbol{V}_f\cdot\nabla T_f\right)=k_f\nabla^2 T_f.
\end{aligned} \tag{2.2}$$

where T_f is the Kelvin temperature of the fluid layer, $\boldsymbol{V}_f$ is the

Fox (1962), Fox & Parker (1968), Orszag (1971a, 1971b) and Orszag & Kells (1980). Chaves & Ortiz (1968) applied it to a second order eigenvalue problem with polynomial coefficients. Bridges & Morris (1984a, 1984b) have given an extensive discussion of the spatial stability problem in the context of the similar boundary layer by using this method. Bukhari (1997) has used this method to solve two-layer problem, and obtained results different of that which Chen & Chen (1988) obtained by using shooting method based on 4th order Runge-Kutta approximation.

Application Of Chebyshev Spectral Methods To Convection Problem In A Horizontal Fluid Layer Bounded By Two Porous Layers

Consider problem of three horizontal layers L_1, L_2, L_3 such that the top of layer L_2 touches the bottom of layer L_1 and the top of layer L_3 touches the bottom of layer L_2. The plane interface between L_2 and L_3 is $x_3 = 0$, and between L_1 and L_2 is $x_3 = d_f$, the upper boundary of L_1 is $x_3 = d_{m1}$ and the lower boundary of L_3 is $x_3 = -d_{m2}$.(see Fig. 1).

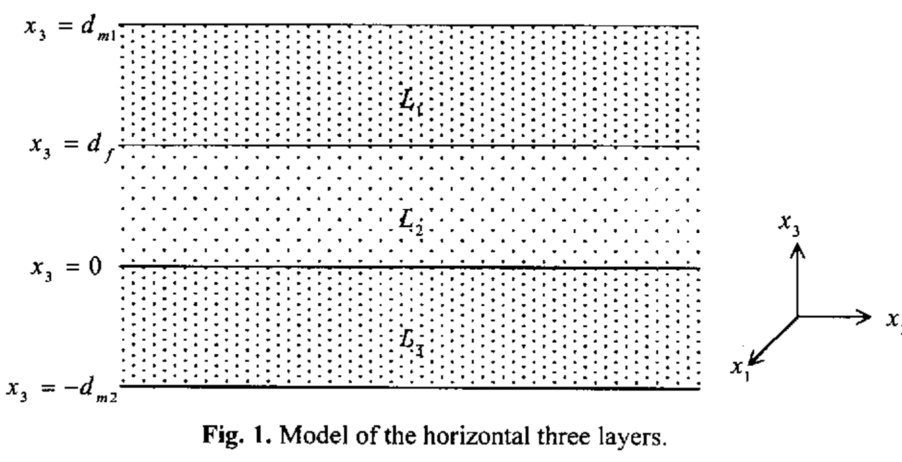

Fig. 1. Model of the horizontal three layers.

methods from the finite-elements methods and finite-difference methods by selection of trial functions.

Generally, the spectral methods classify to several kinds: Galerkin, Collocation, and Tau spectral methods. Differentiate each kind from another by choice suitable test functions. In the first kind "Galerkin", the trial functions and the test functions are the same so the test functions are infinite functions and every one of them satisfy the boundary conditions of differential equation. Silberman (1954) was used this method, it applied to solve non-linear problem by Eliasen et al. (1970) and Orszag (1969,1970). In the second kind "Collocation" it stipulate that the differential equation satisfied at collocation points which are usually zeros of some trial functions. This method used by Slater (1934) and Kantorovic (1934), and developed to solve ordinary differential equations by Frazer et al. (1937), and applied by using Chebyshev polynomials for initial value problems by Clenshaw (1957), Clenshaw & Norton (1963) and Wright (1964), and for boundary value problems by Villadsen & Stewart (1967). The last kind "Tau" is resemble to Galerkin but it does not requisition that the test function satisfy the boundary conditions, hence the boundary conditions are specify by supplementary set of equations. This method developed by Lanczos (1938) and Gardner et al. (1989).

Anyway, these three kinds of the spectral methods are general kinds, but there are also another kinds that we can obtain it by special choice of the trial functions. For example: Chebyshev polynomials and Legendre polynomials that are use on finite intervals, Hermite polynomials that are use on infinite intervals, and Laguerre polynomials when the intervals are semi-infinite. In this work we will apply tau spectral method by using Chebyshev polynomials. This method have used and developed to solve ordinary differential equations by

Chebyshev spectral methods have proven to be a useful technique for supplying accurate efficient answers to linear stability problems. The spectral method that used to solve system of first order differential equations is called *First Order Of Chebyshev Spectral Method* and which used to solve system of second order differential equations is called *Second Order Of Chebyshev Spectral Method.* In this work we compare between the first order and the second order of Chebyshev spectral methods to knowledge which method is more accurate. We use both methods to solve convection problem in three layers.

Introduction

The spectral methods are approximation methods to find the solutions of differential equations (ordinary or partial) for different orders by using expansion of basic function. As special case solves systems of ordinary linear differential equations which tend to compute the eigenvalue of boundary value problem. On the other hand, in fluid dynamics many problems require stability analysis of boundary value problems (i.e. find the eigenvalue). The stability case engenders when the eigenvalue have negative real part whereas the instability case engenders when the eigenvalue have positive real part. Here we have two kinds of instability the first is stationary state when the eigenvalue is real, and the second is overstability state if the eigenvalue is complex. For this reason we use spectral methods to solve fluid dynamics problems numerically.

In fact, the spectral methods known also by weighted residual methods which dependent basically on two kind of functions, that are trial functions (also known as approximation functions, expansion or basic functions) and test functions (or weight functions). Recognition the spectral

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MINISTRY OF HIGHER EDUCATION
Umm Al-Qura University
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CONVECTION IN HORIZONTAL THREE LAYERS COMPARING THE RESULTS OF SOLUTIONS BY FIRST ORDER AND SECOND ORDER OF CHEBYSHEV SPECTRAL METHOD

Dr. Abdul Fattah Q. Bukhari

&

Student. Hanadi Banjer

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